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APPLICATION OF WKB METHOD IN EVALUATION OF ENERGY EIGEN VALUE OF A SYMMETRIC PARABOLIC ALAS QUANTUM WELL STRUCTURE

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ABSTRACT

WKB an acronym for Wentzel- Kramers – Brillouin, is a method employed for finding approximate solution to linear partial differential equations with spatially varying the coefficient. The method was applied in this work to evaluate the energy eigenvalue of AlAs. In determining this, the harmonic potential which is a suitable potential for the system, having an infinite number of energy level, cum the approximate quantization rule (the well-known Bohr-Sommerfeld quantization condition) are used. The WKB method is then tested by the quantization of the quantum mechanical system of the modified harmonic potential which practically gives the same result as using exact methods. Both numerical and graphical results were obtained, and it was observed that the energy value for AlAs symmetric parabolic quantum well increases as the quantum number (n) increases

KEYWORDS: Semiconductors, Wavelength

INTRODUCTION

A number of semiconductor super lattice structures are emerging with wide gap compounds such as AlAs, ZnSe and GaAs as quantum well materials. A quantum well is synthesized from two different semiconductors. The emission energy of the quantum well is different from either band gap energy of the two semiconductors. Thus, a new synthetic material has been formed, that is only remotely related to the bulk material it is made of. This is why quantum wells are exciting materials. These structures will form the basis of the electronic devices of the future. It has been proved that transistors and lasers made of quantum wires demonstrate excellent characteristics. In this work, attempt was made to evaluate Eigen value of a symmetric parabolic AlAS quantum well using WKB method

The WKBf ormular (Wentzel 1926, Kramers 1926, Brilliouin 1926 [Giler, 1988, Merzbacher, 1970], Cocolicchio and Viggiano [Cocolicchio and Viggiano 1997]) provides us with rather simple and interestingly good approximation solution to the schrodinger equation, for this reason it is widely used in many approximation calculations of quantum mechanical systems. [Oyewumi and Bangudu, 1999]

Other forms of the applications of this approximation method is the well known Bohr-Sommerfeld quantization conditions. Oyewumi and Bangudu used WKB method with the derivation of Bohr Sommerfeld quantization conditions to obtaine the discrete eigenvalues (quartic) Oscillator which agrees with Giler's result of 1988 [Giler et al, 1986]. Ghatak et al. applied the JWKB formula to a triangular potential barrier and compare the results with the exact results. Also, Birx and Hourk [Birx and Hourk, 1977] obtained the eigenvalues for the upper and lower bounds of an harmonic oscillator potential

THEORY AND CALCULATION

The WKB Approximation

The Wentzel-Kramers-Brillouin (WKB) method is useful tool for a simple and interestingly good approximate solution of the Schrödinger equation, for this reason, it is widely used in many approximate calculations of quantum mechanical systems. For instance, the treatments of systems with slowly varying potentials; that is, potentials which remain almost constant over a region of the order of the de-Broglie wavelength. Thus, the property is always satisfied since the wavelength of a classical system approaches zero

Consider the motion of a particle in a time-independent potential $V(\vec{r})$; the Schrodinger equation for the corresponding stationary state is

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\psi(\vec{r}) \qquad 1$$

Or

$$\nabla^2\psi(\vec{r})+\frac{1}{\hbar^2}p^2(\vec{r})\Psi(\vec{r})=0 \qquad 2$$

Where p$(\vec{r})$ is the classical momentum at $\vec{r}$. The WKB method provides an approximate treatment for systems whose potentials, while not constants, are slowly varying functions of $\vec{r}$. That is, V $(\vec{r})$ is almost constant in a region which extends over several de-Broglie wavelength; we may recall that the de-Broglie wavelength of a particle of mass m and energy E that is moving in a potential V$(\vec{r})$ is given by

$$\lambda=\hbar/p=h/\sqrt{2m(E-V(\vec{r}))} \qquad 3$$

Thus, we can use WKB method to estimate the energy eigenvalue of AlAs in a symmetric parabolic quantum well with $V_{(x)} = |\frac{1}{2}m\omega^2x^2|$

In common cases of quantum mechanical problems, the harmonic oscillator potential $\frac{1}{2}kx^2$, where k is the force constant of the spring, is used in obtaining eigenvalues and Eigen function analytically in an elegant manner. But when we extend to the harmonic regions: that is, where potentials are involved, analytic solutions are practically impossible to attain. The WKB Method gives an approximate but direct solution of the Schrödinger equation but is applicable only when the potential energy variation is smoothly varying and when the Schrödinger equation is separable to one-dimensional equation

Application of WKB Method to a Symmetric Parabolic Quantum Well

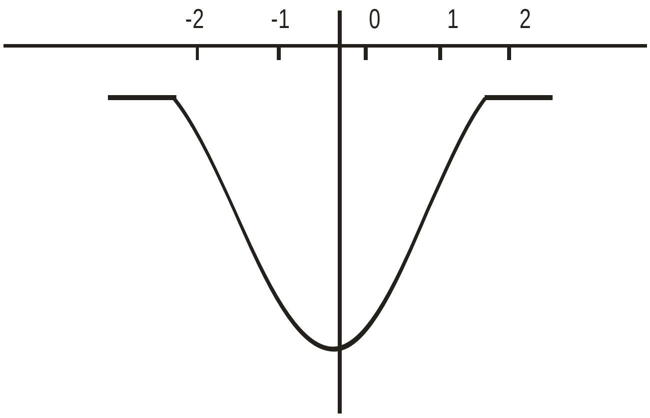

Figure 1: Model of Symmetric Parabolic Quantum Well Used in the Calculation

The classical energy of a harmonic oscillator is given by

$$E(x,p)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2 \qquad 4$$

Leads to

$$p(E,x)=\pm\sqrt{2mE-m^2\omega^2x^2} \qquad 5$$

At the turning points, x_{min} and x_{max}, the energy is given by E = V(x) = $\frac{1}{2}m\omega^2x^2$ where x_{min} =-a and x_{max} = a with a = $\sqrt{2E/(m\omega^2)}$.

To obtain the quantized energy expression of the harmonic oscillator, we used the Bohr-Somerfeld quantization rule

$$\oint pdx = 2\int_{-a}^{a}\sqrt{2mE-m^2\omega^2x^2dx} = 4m\omega\int_0^a\sqrt{a^2-x^2dx} \qquad 6$$

Using the change of variable x=a sin θ, we have

$$\int_0^a\sqrt{a^2-x^2}dx = a^2\int_0^{\pi/2}\cos^2\theta d\theta=\frac{a^2}{2}\int_0^{\pi/2}(1+\cos 2\theta)d\theta=\frac{\pi a^2}{4}=\frac{\pi E}{2m\omega^2}; \qquad 7$$

Hence

$$\oint pdx=\frac{2\pi E}{\omega} \qquad 8$$

Since

$$\oint pdq=(n+\frac{1}{2})hor2\pi E/\omega=(n+\tfrac{1}{2})h, \qquad 9$$

We obtained

$$E_n^{WKB}=(n+\frac{1}{2})\hbar\omega \qquad 10$$

This expression is identical with the exact energy of the harmonic oscillator

RESULTS AND DISCUSSIONS

As evident from the graph (figure), it is revealed that as the quantum number (n) increases, the eigen energy value also increases which is in conformity with the result obtained from other methods of solution

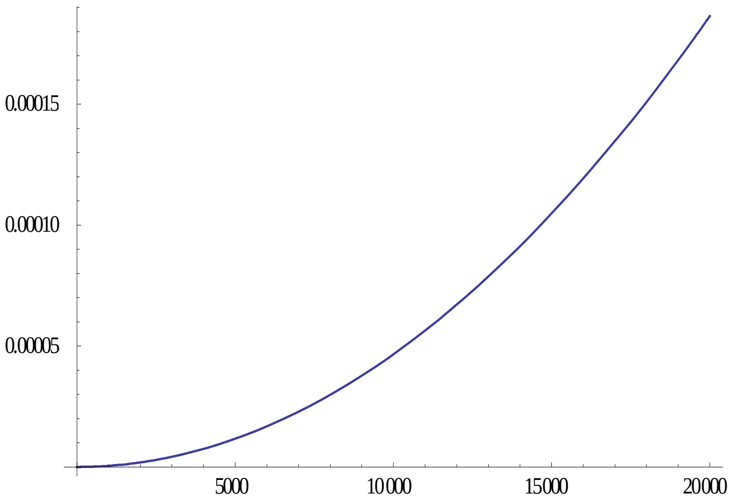

Figure 2: Showing the Relationship between the Quantum Number (N) and the Energy of the System

CONCLUSIONS

Using the WKB method, the result obtain is in agreement with the result of other methods, that is increase in quantum number leads to increase in the energy of the system

The accuracy of the WKB method is best assessed by comparing the WKB discrete energy values of the harmonic potential with those obtained by solving the Schrodinger equation numerically. There are no remarkable difference between the exact eigenvalues and WKB eigenvalues

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